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STABILITY OF THE SPACE CABLE

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ABSTRACT

The space cable consists of evacuated tubes supported by fast-moving objects, called *bolts*, travelling inside them. The bolts are connected to the tubes via permanent magnets stabilized electronically for minimal power consumption. Version 1 was designed to replace the role of a first-stage rocket by lifting space vehicles from the ground to 50 km at 5800 km/hour. Version 2 rises to 140 km and has wider applications. Comparable proposals include the space fountain and the launch loop. In the long term, such fixed infrastructure is more economical than rockets.

A key issue with these dynamically supported structures is lateral stability, particularly in the presence of varying cross winds in the stratosphere. The relevant partial differential equations are derived below with a set of solutions. A proposal is presented for providing stability by means of tethers, pipes, and a support structure at each end. Each support is a miniature space cable rising to 15 km, and it bears the load of the tethers and pipes that would otherwise have to be carried by the main structure.

INTRODUCTION

To escape the earth's gravity well, there are advantages in building a fixed infrastructure rather than using rockets. Among these advantages are:

1. Low cost per journey because there is a much reduced need to carry fuel and reaction mass
2. Low energy consumption and pollution
3. Enablement of gentle rides that allow wide sections of the public to visit space without the need for special training or high fitness

The most famous form of fixed infrastructure proposed is the space elevator.[1] The challenge with that idea is to find a strong enough material capable of forming long threads. An alternative form of fixed infrastructure was proposed in 1980 to overcome this problem.[2] Called the space fountain, it relies on pellets, which are fast projectiles travelling inside evacuated tubes. Coils in the tubes decelerate the rising pellets and accelerate the falling ones, thus causing an upward force that supports the weight of the tubes and of any vehicles. Further work (renamed Starbridge[3]) showed that the energy consumption of the space fountain is unfortunately very high.

Both the space elevator and the space fountain involve a vertical structure reaching to geostationary orbit. Vehicles reach orbit when they attain that height, a process that could take hours or days. Lofstrom[4] proposed the launch loop, which consists of a belt travelling at 14 km/sec, reaching a height of about 80 km and extending over 2000 km of the earth's surface. This is much more efficient than the space fountain at getting vehicles out of the earth's atmosphere and into orbit.

The space cable is smaller than the launch loop and should therefore cost less.[5] The first version was designed to replace the role of first-stage rockets by accelerating a rocket's upper stages to 5800 km/h at a height of 50 km. Another version of the space cable is described here that reaches to near space (140 km) and is suitable for astronomy and other science and also for space tourism. Version 2 is about three times as expensive as the first, but it has many more uses. Figure 1 shows a size comparison.

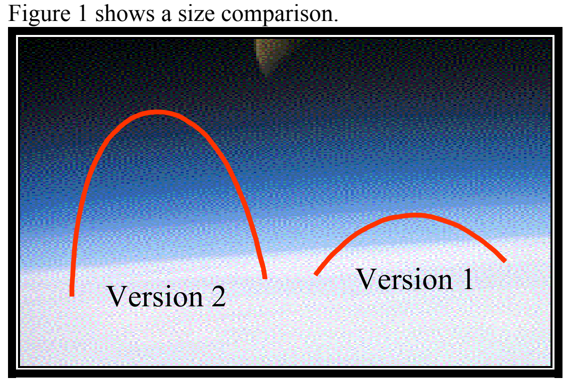

Figure 1 Size comparison between versions

In the space fountain, the launch loop and the space cable, energy and momentum are transferred electromagnetically from the travelling pellets, belt or bolts to the accelerating vehicle. This overcomes the problem in the space elevator of transmitting power to the vehicle.

One of the questions with these proposals is how to achieve stability. The troposphere and stratosphere can be very turbulent, and the winds are strong. The launch loop requires tethers to be attached at certain points and anchored to the earth's surface to stabilize it. One problem with this approach is that the tethers cause substantial extra tension in the space cable, and this leads to greatly increased forces and costs. This problem can be overcome by erecting a pair of support structures, which are miniature space cables, to a height of 15 km. This is an adaptation of a means of deflecting the space cable at the surface stations, described in Version 1. That work did not deal with the oscillation modes. Now, the oscillations are examined. We present equations for this motion and show how support, tethers and pipes can provide the necessary stability.

SPACE CABLE VERSION 1

The space cable consists of several pairs of evacuated tubes in which fast-moving projectiles, known as *bolts*, support the weight. The tubes have a diameter of 5 cm and, in version 1, reach a height of 50 km while covering a distance on the ground of 150 km. Magnetic levitation is used to maintain a practically frictionless connection between the bolts and the tubes. At each end there is a surface station that has the job of initially accelerating the bolts and then, when they return, of turning them around again. The surface stations may be on the ground or at sea.

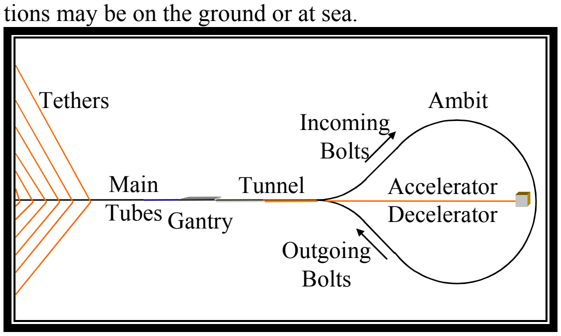

Figure 2 Plan view of surface station with ambit

At each surface station, there is a gantry and tunnel (Figure 2). They turn the descending bolts to the horizontal and pass them to a circular arrangement of superconducting magnets called the *ambit*. The radius of the ambit is 330 metres, and the bolts' velocity is up to 2.5 km/sec. The bolts proceed back along the ramp and up into a tube at an angle of 56° to the horizontal. The tubes are paired: one carries descending and one carries ascending bolts.

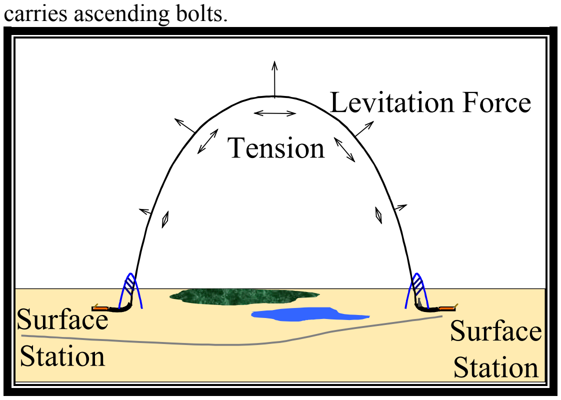

Figure 3 Space cable shape and distribution of forces

As illustrated in Figure 3, the shape of the space cable resembles that of an inverted catenary; the exact equation is given in reference 5. Magnetic levitation between a tube and the bolts inside it causes a force perpendicular to the bolts' direction of travel. This force supports the tube and consequently lowers the bolts' trajectory as if the weight of each bolt had increased. In the lower parts of the tube, the force is at an angle and only supports part of the tube's weight. The rest of the weight is sustained by tension in the tube, which transfers the force to the top. At the top, the bolts are travelling nearly horizontally and support the whole weight of the tube there. They also support part of the weight of the lower parts of the tube. Figure 3 shows the relative forces at different points.

This arrangement, whereby the forces are perpendicular to the bolts' motion, is different from that of the space fountain, where bolts have to be retarded or accelerated to support the tubes. This is how the space cable avoids the heavy losses caused by heating of the coils in the space fountain.

The space cable makes further substantial energy savings by using a novel application of permanent magnets to magnetic levitation. Permanent magnets have been demonstrated successfully in magnetic bearings[6] and in a prototype train technology known as Inductrack.[7] The space cable takes this method further to minimize the losses due to eddy currents, as described in reference 5.

SPACE CABLE VERSION 2

Version 1 was aimed at reducing the cost of launching space vehicles. Version 2 reaches almost three times as high – to 140 km – which is near space rather than in the upper atmosphere. Version 2 can still be used for launching space vehicles, but it is high enough to enable other applications. Many earth satellites are placed in orbit for scientific work, the most famous being the Hubble space telescope. However, at 140 km the space cable can provide a platform for science with many advantages over satellites. The biggest advantage is that instruments can be raised or lowered easily for upgrade and maintenance; they can even be accessed in place. Consider how beneficial that would have been when errors were discovered in the Hubble's mirror. The cost of producing such instruments can be drastically lowered if they can be accessed readily after deployment.

Another potential revenue earner is tourism. Like the space elevator, the space cable can transport people to near space at a gentle 150 km/hour, making the experience accessible to a wide public. For those customers wanting the thrill of weightlessness and reentry, it is quite feasible to build vehicles that can drop back into the atmosphere and glide or fly home. The reentry velocity into the atmosphere from being stationary at 140km is about 3000 km/h – much less violent than reentry from orbit.

Version 2 achieves the greater height by doubling the bolts' mass to 10 kg and increasing their velocity to 3.4 km/sec. The ambit radius becomes 600 metres. The angle of inclination at the surface stations is 74°.

The greatest change is the tension in the tubes. The preferred material is Kevlar®*, because it is widely used and reasonably priced. In version 1, up to 4.5 kg of Kevlar® per metre is needed to sustain the tension, but this increases to 19 kg per metre at the top in version 2. In both cases, a factor of four is included as a safety margin, as is customary engineering practice.

The height of 140 km is carefully chosen; above this height, the exponential factor in equation (3) (see section "Calculations") kicks in more severely. At 200 km, for example, 46 kg per metre would be needed with a bolt velocity of 10 km/sec, leading to substantial increases in ambit size and other costs. Another advantage of the altitude of 140 km is that it is below the level at which space debris is known to orbit. Objects orbiting at this height fall to earth rapidly.

Calculations

The equations are similar to those previously reported except for two changes. In version 1, a constant tube weight per metre was assumed that was sufficient to sustain the maximum tension. In version 2, the tube weight varies by a factor of 10 and must be calculated explicitly. Furthermore, the additional height makes it worth taking into account the reduced gravity, which was neglected in version 1.

Define $\alpha=4\rho/S$, where $\rho=1.47\times10^3$ kg/m^3 is the density of Kevlar®, S=3450 MegaPascal (MPa) is its strength, and 4 is the safety factor. The weight w of the tube per metre is given by

$$w=(\alpha T+m_t)g \qquad (1)$$

Here m_t is the mass of the tube due to magnets and other materials, g is the acceleration due to gravity, and T is the tension, which also satisfies

$$T=\int_{r_0} w\,dr+T_0 \qquad (2)$$

Here T_0 is the tension at ground level, r is the distance from the centre of the earth, and r_0 is the value of r at ground level. The solution to these equations is

$$T=\left(T_0+\frac{m_t}{\alpha}\right)e^{\alpha(r-r_0)g}-\frac{m_t}{\alpha} \qquad (3)$$

and

$$w=(\alpha T_0+m_t)g e^{\alpha(r-r_0)g} \qquad (4)$$

* Kevlar® is a Dupont registered trademark. See MatWeb Material Property Data at URL www.matweb.com

If g_0 is the acceleration due to gravity at ground level, then $g=g_0 R_0^{\,2}/r^2$. These equations lead to the differential equation

$$\ddot{r}=\frac{r\dot{\theta}^2-g-Cr\dot{\theta}\left[(m_t+\alpha T)g\,r\dot{\theta}/V-qT\right]}{1-Cr\dot{\theta}uT} \qquad (5)$$

As in version 1, it has the form

$$\ddot{r}=f(\dot{r},r) \qquad (6)$$

This can be solved numerically in the same way using the Runga-Kutta method.

In equation (5), a dot denotes differentiation with respect to time. The angle θ is that subtended at the centre of the earth in plane polar coordinates. The constant C is given by $C=s_0/m_bV_0$, where s_0 is the spacing between bolts at ground level, m_b is the mass of a bolt, and V_0 is the velocity of a bolt at ground level. The symbols q and u are given by:

$$q=\frac{1}{V^3}\frac{g}{r\dot{\theta}} \qquad (7)$$

and

$$u=\frac{1}{V^3}\left(r\dot{\theta}+\frac{\dot{r}^2}{r\dot{\theta}}\right) \qquad (8)$$

We also have

$$\dot{\theta}^2=\frac{V^2-\dot{r}^2}{r^2} \qquad (9)$$

and

$$V^2=V_0^2-2g(r-R_0) \qquad (10)$$

STABILITY

A wind blowing across the space cable will deflect it. If the winds were predictable and steady, the surface stations could compensate by deflecting the cable in the opposite direction. However, winds are often gusty or turbulent, and this leads to oscillations. Analysis of the equations (section "Derivation of the Stability Equations") shows that there are solutions that involve waves that travel along the tubes at the velocity of the bolts, but there are also solutions with positive and negative exponential terms. The positive exponential terms often dominate, which means that, once the space cable has bent beyond a certain curvature, its rate of bending increases indefinitely.

At heights above about 12 km, the air is still. The problems occur below that height, particularly when there are strong jet-stream winds. In the launch loop, the proposed solution is to fix tethers from the cable down to the ground. However, this places considerable extra weight on the overall structure, requiring substantially thicker cables because of the exponential law in equation (3). In version 2 of the space cable, the effect of this extra weight is to double the thickness of Kevlar® needed at the top, substantially increasing the bolt velocity and hence the ambit radius, and requiring over four times the permanent-magnet strength in the upper parts of each tube.

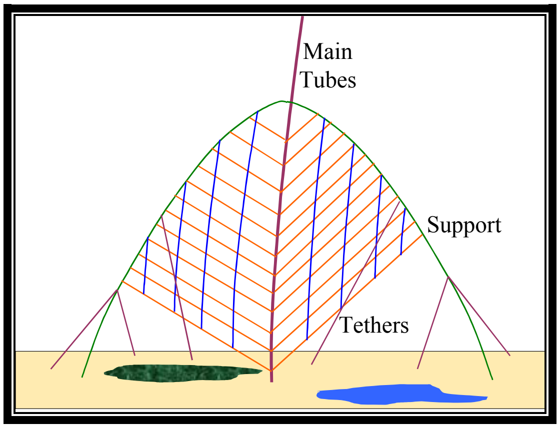

Figure 4 Impression of support and tethers

A more economical solution is to erect a support structure near each surface station (Figure 4). Each support is a space cable in miniature; it stands at right angles to the main space cable. The main space cable is tethered to the supports, not to the ground. Each support's height of 15 km is designed so that the tethers and support carry the weight of the space cable up to this height. They also carry the weight of aluminium pipes provided to give extra stiffness. Some tethers are at 45° while others are vertical.

The support has to be tethered to the ground, and it carries the weight of its own tethers, but this avoids the cumulative effect of transmitting the weight to the top of the space cable. The details are in the next sections.

<u>Derivation of the Stability Equations</u>

To understand the effect of external forces (mainly wind) on the space cable, we can derive partial differential equations as follows.

Consider two points A and B close together on a curved tube (Figure 5). If a bolt reaches A at time t, it reaches B at time $t+\delta t$. Then the distance between A and B is

$$\delta x = V\delta t \qquad (11)$$

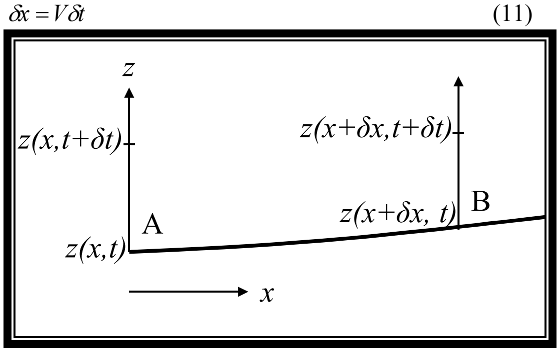

Figure 5 Displacements as a bolt travels from A to B

Between A and B, the displacement of the bolt is

$$z(x+\delta x,t+\delta t)-z(x,t)$$
$$\approx z(x+\delta x,t)-z(x,t)+z(x,t+\delta t)-z(x,t)$$

Hence

$$\frac{z(x+\delta x,t+\delta t)-z(x,t)}{\delta t} \approx V\frac{z(x+\delta x,t)-z(x,t)}{\delta x}+\frac{z(x,t+\delta t)-z(x,t)}{\delta t} \qquad (12)$$

In the limit, this equation becomes

$$\frac{dz}{dt}=V\frac{\partial z}{\partial x}+\frac{\partial z}{\partial t} \qquad (13)$$

Similarly,

$$\frac{d^2z}{dt^2}=V\frac{\partial}{\partial x}\left(V\frac{\partial z}{\partial x}+\frac{\partial z}{\partial t}\right)+\frac{\partial}{\partial t}\left(V\frac{\partial z}{\partial x}+\frac{\partial z}{\partial t}\right) \qquad (13)$$

This leads to the equation for a bolt's acceleration

$$\frac{d^2z}{dt^2}=\frac{\partial^2 z}{\partial t^2}+2V\frac{\partial^2 z}{\partial x\partial t}+V^2\frac{\partial^2 z}{\partial x^2} \qquad (14)$$

A pair of tubes is connected by damped elastic struts positioned every u metres. If a bolt's mass is m_b and their separation is s, it is convenient to consider an average bolt mass m_b per u metres. Then the force at a strut caused by the bolts' acceleration in one tube is

$$m_b\left(\frac{\partial^2 z}{\partial t^2}+2V\frac{\partial^2 v}{\partial x\partial t}+V^2\frac{\partial^2 z}{\partial x^2}\right) \qquad (15)$$

To this must be added the force due to the tube's acceleration $m_t\dfrac{\partial^2 z}{\partial t^2}$.

Writing $m_u=m_b+m_t$, the force F_a due to acceleration of the bolts and the tube is given by

$$F_a=m_u\frac{\partial^2 z}{\partial t^2}+2m_bV\frac{\partial^2 v}{\partial x\partial t}+m_bV^2\frac{\partial^2 z}{\partial x^2} \qquad (16)$$

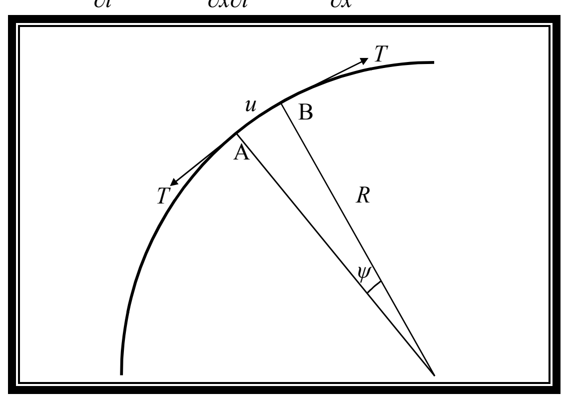

Figure 6 Contribution of tension

Balancing F_a are three more forces, an external force F generally due to wind, a force F_s in the strut and the tension T in the tube. The contribution of the tension (Figure 6) over the angle ψ between points A and B distance u apart is $T\sin\psi$. For small u compared with the radius of curvature R, we have a force

$$-T\sin\psi \approx -T\psi \approx -Tu/R$$

Then the force due to tension at a strut is given by

$$F_T = -T\frac{u}{R} = Tu\frac{\partial^2 z}{\partial x^2} \qquad (17)$$

The balance of forces at one end of a strut is given by

$$F_a = F_T + F + F_s \qquad (18)$$

A strut connects two tubes in which the bolt velocities are equal and opposite. Writing z_1 for the z-displacement in one tube and z_2 for the other, we combine equations (16), (17) and (18) to obtain the following pair of simultaneous partial differential equations for a pair of tubes connected by a strut

$$m_u\frac{\partial^2 z_1}{\partial t^2} + 2m_b V\frac{\partial^2 z_1}{\partial x \partial t} + \left(m_b V^2 - Tu\right)\frac{\partial^2 z_1}{\partial x^2} = F + F_s \quad (19)$$

$$m_u\frac{\partial^2 z_2}{\partial t^2} - 2m_b V\frac{\partial^2 z_2}{\partial x \partial t} + \left(m_b V^2 - Tu\right)\frac{\partial^2 z_2}{\partial x^2} = F - F_s \quad (20)$$

The first term in these equations is the straightforward acceleration of the tubes and bolts, independent of the motion of the bolts. The third term represents an effect due to curvature, moderated by tension. Once a tube starts to bend, the bolts' momentum tends to resist turning, thus exerting an outward force that causes the bend to increase indefinitely. The second term in equations (19) and (20) represents a propagation effect carried along by the bolts at velocity V. If the struts are rigid, then $z_1 = z_2 + L$, and there is no propagation. However, with elasticity and damping in the struts, a degree of play is possible. The overall effect is that the bending of the tubes due to a localized wind is spread along the tubes in both directions rather than being concentrated in one place. This slows the rate of bending and reduces the acuteness of the instability.

If L is the nominal length of a strut, ε is its expansion coefficient, and p is the damping coefficient, the force in a strut is given by

$$F_s = -\left(\frac{\partial z_1}{\partial t} - \frac{\partial z_2}{\partial t}\right)p - \left(z_1 - z_2 - L\right)\varepsilon \qquad (21)$$

When F=0, the equations have solutions of the form

$$z_1 = Ae^{\alpha t}\left(e^{\gamma \alpha x/V} + Be^{-\gamma \alpha x/V}\right) + \tfrac{1}{2}L \qquad (22)$$

$$z_2 = \pm Ae^{\alpha t}\left(e^{-\gamma \alpha x/V} + Be^{\gamma \alpha x/V}\right) - \tfrac{1}{2}L \qquad (23)$$

These satisfy the physical expectation of symmetry, given the opposing velocities in a pair of tubes. We can solve for B and α, leaving A and γ as free variables. One form of the solution leads to travelling waves at the bolts' velocity, but the real exponential terms require further examination. Write

$$M = \tfrac{1}{2}\left(m_u + (m_b - Tu/V^2)\gamma^2\right) \qquad (24)$$

This leads to the following equations for B and α:

$$B = \pm\frac{M - m_b\gamma}{M + m_b\gamma} \qquad (25)$$

and

$$M\left(M\alpha^2 + p\alpha + \varepsilon\right) - m_b^2\gamma^2\alpha^2 = 0 \qquad (26)$$

This quadratic equation in α has the solution:

$$\alpha = \frac{-Mp \pm \sqrt{M^2 p^2 - 4M\varepsilon(M^2 - m_b^2\gamma^2)}}{2(M^2 - m_b^2\gamma^2)} \qquad (27)$$

The quantity γ is a free variable, which may be complex. Therefore, we can make no assumptions about the sign of the real part of α. Since real positive values of α are possible, these will tend to dominate, leading to exponential growth in the deformation of the space cable when winds disturb it. For this reason, the supports described in section "Support Structure" are proposed.

<u>Support Structure</u>

In equations (19) and (20), the dominant factor tends to be $m_b V^2 \frac{\partial^2 z}{\partial x^2}$. The greater the curvature of a tube, the more centrifugal force is created as bolts travel round the curve at velocity V. Previous calculations (reference 5) have shown that a likely maximum wind force on a tube is about 50N. The introduction of an aluminium pipe increases this to about 100N. The arrangement of tethers and supports needs to cope with this force plus the centrifugal effect, but that effect needs to be limited by adding stiffness to the tubes at the elevations at which winds occur, typically up to 12 km. 15 km is taken as a safer working assumption.

The added stiffness is provided by attaching an aluminium pipe to each tube comprising the space cable. The proposed mass of this *stiffening pipe* is 40 kg/metre, as shown in section "Calculations for Pipes."

Fine tethers that are closely spaced are more effective than coarse tethers widely spaced. The wider the spacing, the greater is the reliance placed on the stiffening pipe. A pair of tethers every metre is proposed, at an angle of 45° to the tubes, with one on the right and one on the left. These are *sloping tethers*, and they rise from the main cable to the support. The combination of tethers and the support hold the weight of the stiffening pipes and also provide lateral stability. The proposed material for the tethers is Kevlar®. The average weight of a pair of sloping tethers is 280 N (see section "Calculations for Tethers"). Because of the angle of the support, the average spacing of these is about 1.2 metres, giving an average load per metre of 240 N or nearly 24 kg.

To minimize sagging of the sloping tethers, *vertical tethers* are placed at 1 km intervals along each sloping tether. Overall, the tethers form a network, with the vertical tethers bearing the weight of the sloping tethers. The load per metre of the vertical tethers works out at 42 N (see section "Calculations for Tethers").

The net effect of the arrangements of tethers is to keep the movement under strong winds below 2.5 metres (see section "Expansion and Displacement").

There is a support at each end of the space cable, and they are themselves space cables in miniature. Each support is stabilized by being tethered to the ground. In addition to bearing the load of the stiffening pipes and tethers, it must bear the forces in these ground-attached tethers. These forces are in balance, and so the effect of the ground-attached tethers is to double the load on the support. Furthermore, the support requires its own stiffening pipes at 40 kg/metre. The net effect is to place a load of 140 kg/metre on the support. Using the methods of the section "Space Cable Version 2", this leads to a bolt velocity in the support of 3.1 km/sec. The ambit radius for the support is 500 metres. The support rises to 15 km and also extends over a horizontal range of 15 km.

The next section gives the details by which these numbers are calculated.

Calculations for Tethers

Each pair of sloping tethers bears the weight of a metre length of pipe, about 40 kg, amounting to nearly 200 N on each tether. In the mid position, they also bear equal and opposite tensions of 200 N, giving a total tension of 200√2, or nearly 300N. Under cross winds, they will move laterally, so that either of them may bear nearly the whole load, up to 400 N horizontally and the same vertically. This increases the total tension from about 300 N to about 600 N.

The weight w of a tether of height h required to support tension T satisfies

$$w = \frac{4\rho g(T + wh)}{S} \tag{28}$$

Here, S is the strength (3450 MPa), ρ is the density (1.47×10^3 kg/m^3), g is the acceleration due to gravity, and 4 is a safety factor. Hence $w=1.4\times10^{-2}$ N/metre for a sloping tether. Hence the weight of an average 10 km tether is 140 N or 280 N per pair, as stated in section "Support Structure".

A vertical tether of a typical 10 km height supports 10,000 sloping tethers, giving a maximum tension of 1.4×10^5 N, requiring a weight of 3 N/metre or 3×10^4 N altogether. The vertical tethers are spaced along the support at intervals of approximately 700 metres, and so the average load per metre on the support is 42 N, as stated in section "Support Structure".

Expansion and Displacement

As the tension increases, a tether expands according to the formula

$$e = \frac{Tl}{\sigma E} \tag{29}$$

Here, e is the extension in metres, l is the length without tension, $\sigma=w/\rho$ is the area of cross section, and E is the modulus of elasticity (179 Gpa). For a tension increasing from 300 N to 600 N in a cross wind, the extension of a sloping tether comes to 1.7 metres.

A sloping tether forms a catenary, and increased tension changes the geometry, leading to further movement that must be added to the extension. If χ_A and χ_B are the horizontal displacements at the top and bottom of the catenary, and H is the horizontal component of the tension, then the length l and height h satisfy[8]

$$l = \frac{H}{w}\left(\sinh\frac{w\chi_A}{H} - \sinh\frac{w\chi_B}{H}\right) \tag{30}$$

and

$$h = \frac{H}{w}\left(\cosh\frac{w\chi_A}{H} - \cosh\frac{w\chi_B}{H}\right) \tag{31}$$

Using standard equalities for hyperbolic functions including

$$\cosh^2\left(\frac{w}{2H}(\chi_A + \chi_B)\right) - \sinh^2\left(\frac{w}{2H}(\chi_A + \chi_B)\right) = 1 \tag{32}$$

we obtain

$$\sinh\left(\frac{w}{2H}(\chi_A - \chi_B)\right) = \frac{w}{2H}\sqrt{l^2 - h^2} \tag{33}$$

The presence of the vertical tethers enables us to consider each length of 1km separately. Since $l \approx h\sqrt{2}$ at 45° slope, the value of the right-hand side when H=200 is approximately 0.02. If the tension doubles, this value reduces to 0.01. If χ'_A and χ'_B are the equivalent displacements when H doubles, we obtain

$$\frac{\chi'_A - \chi'_B}{\chi_A - \chi_B} = 2\frac{\sinh^{-1} 0.02}{\sinh^{-1} 0.01} \approx 1.00005 \tag{34}$$

This gives a horizontal displacement of 0.5 metre over a typical 10 km height.

Added to the expansion effect of equation (29), the total movement under the increased tension is under 2.5 metres, as stated in section "Support Structure".

Calculations for Pipes

The final consideration for stability is the force on a tube caused by its curvature as bolts travel through it at velocity V. As in equation (19), this is

$$F_c = m_b V^2 \frac{\partial^2 z}{\partial x^2} \tag{35}$$

This force is added to the external force F_w due to wind to give $F=F_c+F_w$.

Now the deflection of a tube of length K under a load of F Newtons per metre is given by[9]

$$EI\frac{\partial^2 z}{\partial x^2} = \frac{Fx^2}{2} - \frac{FKx}{2} \tag{36}$$

Here, I is the moment of area. For a tube of inner and outer radii r_1 and r_2, I is given by

$$I = \frac{\pi(r_1^4 - r_2^4)}{8} \tag{37}$$

Combining (35) and (36), we can obtain an overall force per metre for a length K of tube between tethers by integrating F_c to give

$$F_c' = \frac{m_b V^2}{2KEI}\int_0^L \left(Fx^2 - FKx\right)dx = \frac{FK^2 m_b V^2}{12EI} \quad (38)$$

Previous calculations (see section "Support Structure") showed the maximum force in a jetstream wind to be about 100 N. The tether calculations were based on a maximum lateral force of 400 N, a ratio of 4:1, so we want to set

$$F = F_c' + F_w = 4F_w \quad (39)$$

Combining equations (39) and (38) gives

$$r_1^{\,4} - r_2^{\,4} = \frac{8m_b V^2 K^2}{9\pi E} \quad (1.1)$$

Taking E=7×10^{10} Pa for aluminium, K=1 metre, r_2=4 cm, and V and m_b as before (V=3.4×10^3 m/sec, m_b=10 kg), we obtain an outer radius r_1 of 8.5 cm. Since the mass per metre is $\rho\pi(r_1^2 - r_2^2)$, and ρ=2.6×10^3 kg/m^3, we obtain a mass per metre of 40 kg, as stated in the section "Support Structure".

CONCLUSION

This approach to the problem of stability in the space cable is rather heavyweight. It only deals with lateral stability, which is the most significant problem. Overall, the space cable is vertically stable because of the influence of gravity, but vertical oscillations still need to be accounted for.

In principle, with a constant cross wind, it is possible to align the space cable so that its curvature exactly counteracts that cross wind; this means deflecting the cable towards the wind so that the wind pushes it back into line. However, such stable conditions are rare, and it has not proved possible to devise a dynamic scheme to take advantage of this idea.

Another area of concern is the possibility of perturbations above the level of the proposed tethers and supports. As there is little air movement at these altitudes and the air is very thin, there is less of a problem. However, over the long term, some disturbances are likely at very high altitudes, and a suitable means must be devised to deal with them.

This report presents work in progress. Further investigation may lead to other solutions.

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